Scissors Modes: The elusive breathing overtone

Keisuke Hatada[1,2], Kuniko Hayakawa[2], Fabrizio Palumbo[2]
[1] *Scienze e Tecnologie, Università di Camerino,*
Via Madonna delle Carceri 9, 62032 Camerino, Italy,
[2] *INFN Laboratori Nazionali di Frascati, 00044 Frascati, Italy*
(Dated: April 19, 2013)

The Two-Rotor Model predicts two levels above the Scissors Modes with degenerate intrinsic energy. They have $J^\pi = 0^+, 2^+$ and are referred to as overtones. Their energy is below threshold for nucleon emission, which should make them observable. The $J^\pi = 0^+$ overtone, that has the structure of an isovector breathing mode, has vanishing $E0$ amplitude so that cannot be directly excited, but it could be reached in the decay of the $J = 2^+$ overtone. We discuss such a process and evaluate the $B(E2)$ strength, which, however, turns out to be very small .

PACS numbers: 24.30.Cz,24.30Gd,21.10.Re, 21.60.Ev

The Two-Rotor Model, which led to the prediction of the Scissors Modes [1], has approximately the spectrum of a planar harmonic oscillator, with some constraints on the states. There are two Scissors Modes with $J^\pi = 1^+, 2^+$. They differ by the rotational energy of the nucleus, but have the same intrinsic energy of the order of $3\,MeV$ in the rare earth region [2,3]. The first levels above the Scissors Modes, referred to as overtones, have $J^\pi = 0^+, 2^+$ and equal intrinsic energy of the order of $6\,MeV$. In general one cannot expect that the higher lying excitations of semiclassical models are realized in Nature, the more so for the Two-Rotor Model, because of the modest collectivity and substantial fragmentation of the Scissors Modes [2,3]. Because the energy of the overtones, however, is below threshold for nucleon emission, their widths should remain of purely electromagnetic nature, and might be small enough for these overtones to be observable.

The collective motion of the $J = 0^+$ overtone is similar to that of an isovector breathing mode. Its theoretical existence was known since the beginning [1], but did not rise any interest because within the Two-Rotor Model it has vanishing $E0$ amplitude (but this amplitude might be finite in a description in terms of surface vibrations).

The structure of the $J = 2^+$ overtone was not determined theoretically in the framework of the Two-Rotor Model until recently [4] (even though its possible occurrence was considered in Ref.[5]), because its excitation amplitude was expected to be too small. Indeed excitation amplitudes in the Two-Rotor Model are proportional to powers of θ_0, the amplitude of the zero point oscillation, which in the rare earth region is of order 10^{-1}. Now $B(M1)\uparrow_{scissors} \sim 1/\theta_0^2$, but $B(E2)\uparrow_{scissors} \sim \theta_0^2$, the strength which was expected for the $J = 2$ overtone. All the other methods, the schematic Random Phase Approximation [6], the Interacting Boson model [7], the sum rule method [8] and a geometrical model [9] give similar results for the Scissors Modes.

Recently, nevertheless, we decided to study the $J = 2^+$ overtone within the Two-Rotor Model, and we found [4] that the dominant term of its $B(E2)$ is of zero order in the expansion with respect to θ_0, and precisely

$$B(E2, 0^+ \to 2_1^+) = \frac{3}{64} e^2 Q_{20}^2 \tag{1}$$

where the 2^+ overtone is denoted 2_1^+ to distinguish it from the 2^+ Scissors Mode (see the table). This value must be compared with

$$B(E2)\uparrow_{scissors} = B(E2, 0^+ \to 2^+) = 3\theta_0^2 e^2 Q_{20}^2\,. \tag{2}$$

The numerical factor in Eq.(1) largely compensates for the presence of the small factor θ_0^2 in Eq.(2), but the strength of the overtone remains almost a factor of 2 larger than that of the $J = 2^+$ Scissors Mode in the rare earth region.

Also the $B(E2)$ for the decay of the $J = 2^+$ overtone to the isovector breathing mode is of zero order in θ_0. This suggests that we might reach the breathing mode through the decay of $J = 2^+$ overtone. This process should occur via radiation of photons of energy

$$E = \frac{3\hbar^2}{\mathcal{I}} \tag{3}$$

where $\mathcal{I}$ is the moment of inertia of the nucleus. It is then interesting to study this decay. Firstly, in order to establish whether the isovector breathing mode can actually be seen. Needless to say, its observation in spite of the modest collectivity and substantial fragmentation of the Scissors Modes, would be of much consequence for the picture of Scissors Modes provided by the Two-Rotor Model, the more so in view of the very specific structure of the $J = 2^+$ overtone. Secondly, existence, but also absence, of such a decay might be relevant to the identification of the $J = 2^+$ overtone. In any case the evaluation of the $B(E2)$ for such a process is necessary for the knowledge of the total electromagnetic width of this level. A further reason of interest is related to the other electrically charged systems for which Scissors Modes have been predicted [11].

In the present paper we report the evaluation of the $B(E2)$ strength. To our surprise, at variance with the

$B(E2)\uparrow_{overtone}$, a small numerical factor due a cancellation that appears totally accidental, overcompensates the absence of the small factor θ_0^2, yielding a negligible strength.

In order to make the paper a minimum self contained we report the relevant features of the Two-Rotor Model. Its hamiltonian is

$$H = \frac{\vec{I}^2}{2\mathcal{I}} + H_{intr} \tag{4}$$

where the first term is the total rotational energy of the nucleus, $\vec{I}$ being the total angular momentum and the second one the intrinsic Hamiltonian which, in the slightly modified form of Ref. [10], neglecting terms of order θ_0 is

$$\begin{aligned} H_{intr} &= \frac{1}{2\mathcal{I}}\Bigg[-\frac{d^2}{d\theta^2} - \big(2+\cot^2(2\theta)\big) \\ &+ \cot^2\theta\, I_\zeta^2 + \tan^2\theta I_\eta^2\Bigg] + V(\theta)\,. \end{aligned} \tag{5}$$

θ is half the angle between the proton neutron axes, I_ζ their bisector, $I_\xi, I\eta$ the other orthogonal axes of the body-fixed frame and V the proton-neutron interaction potential. The range of θ can be separated into two regions

$$s_I = s(\theta)s\left(\frac{\pi}{4}-\theta\right),\; s_{II} = s\left(\frac{\pi}{2}-\theta\right)\, s\left(\theta-\frac{\pi}{4}\right), \tag{6}$$

where $s(x)$ is the step function: $s(x) = 1, x > 0$ and zero otherwise. They are obtained from each other by the reflection of θ with respect to $\pi/2$. It is convenient to introduce the notation

$$R_\theta f(\theta) = \mathring{f}\,(\theta) \tag{7}$$

where

$$\mathring{f}\,(\theta) = f\left(\frac{\pi}{2}-\theta\right), \tag{8}$$

so that $\mathring{s}_I = s_{II}$. We assume $\mathring{V} = V$, as appropriate to the geometry of the system. Since we know that the angle between the neutron-proton axes is very small we can assume for the potential a quadratic approximation

$$V = \frac{1}{2}C\,\theta_0^2\,x^2 s_I + \frac{1}{2}C\,\theta_0^2\,y^2 s_{II} \tag{9}$$

where

$$\theta_0 = (\mathcal{I}C)^{-\frac{1}{4}}\,,\;\; x = \frac{\theta}{\theta_0}\,,\;\; y = \frac{\frac{\pi}{2}-\theta}{\theta_0}\,. \tag{10}$$

The intrinsic hamiltonian is then invariant with respect to the transformation

$$R = R_\xi\left(\frac{\pi}{2}\right)R_\theta \tag{11}$$

where R_ξ is the rotation operator around the ξ-axis, so that we can study the eigenvalue equation separately in the regions I, II. The eigenfunctions and eigenvalues of H_{intr} in region I are [10]

$$\varphi_{Kn}(x) = \sqrt{\frac{n!}{(n+K)!\,\theta_0}}\; x^{K+\frac{1}{2}}\, L_n^K\left(x^2\right) e^{-\frac{1}{2}x^2} \tag{12}$$

$$\epsilon_{nK} = \omega(2n+K+1)\,,\;\; \omega = \sqrt{\frac{C}{\mathcal{I}}} \tag{13}$$

where the L_n^K are Laguerre polynomials and the wave functions φ are normalized according to

$$\int_0^\infty dx\;(\varphi_{Kn}(x))^2 = \frac{1}{2}\,. \tag{14}$$

Enforcing the symmetries of the Hamiltonian and of the system (separate rotations of the proton and neutron bodies through π around the ξ-axis), one finds the following results [1],[10]. Restricting ourselves to states of positive parity we found

$$\Psi_{IM\sigma} = \sum_{K\geq 0}\mathcal{F}^I_{MK}(\alpha,\beta,\gamma)\Phi_{IK\sigma}(\theta) \tag{15}$$

where

$$\mathcal{F}^I_{MK} = \sqrt{\frac{2I+1}{16(1+\delta_{K0})\pi^2}}\left(\mathcal{D}^I_{MK} + (-1)^I\mathcal{D}^J_{M-K}\right). \tag{16}$$

I, M are the nucleus angular momentum and its component on the z-axis of the laboratory system, and σ labels the different states with the same I. When it is zero it will be omitted. We will make an assignment of this quantum number different from previous papers, in which we ignored the $J = 0^+$ overtone. We impose the normalization

$$\int_0^{2\pi} d\alpha \int_0^{\pi} d\beta \int_0^{2\pi} d\gamma \int_0^{\frac{\pi}{2}} d\theta\, |\Psi_{IM\sigma}|^2 = 1\,. \tag{17}$$

Secondly the expressions of the intrinsic functions Φ in regions I and II are related according to

$$\begin{aligned} \Phi_{00} &= \varphi_{00}\, s_I + \mathring{\varphi}_{00}\, s_{II} \\ \Phi_{11} &= \Phi_{210} = \varphi_{10}\, s_I - \mathring{\varphi}_{10}\, s_{II} \\ \Phi_{001} &= \varphi_{01}\, s_I + \mathring{\varphi}_{01}\, s_{II} \\ \Phi_{201} &= \frac{1}{\sqrt{2}}\left[\varphi_{01}\, s_I - \frac{1}{2}\left(\sqrt{3}\,\mathring{\varphi}_{20} + \mathring{\varphi}_{01}\right) s_{II}\right] \\ \Phi_{221} &= \frac{1}{\sqrt{2}}\left[\varphi_{20}\, s_I + \frac{1}{2}\left(\mathring{\varphi}_{20} - \sqrt{3}\,\mathring{\varphi}_{01}\right) s_{II}\right]. \end{aligned} \tag{18}$$

Even if the nucleus in its ground state has axial symmetry, this symmetry is in general lost in the excited states, so that the component of angular momentum along any body-fixed axis is not conserved, resulting in a superposition of intrinsic states with different K

quantum numbers	J^π_σ	K	n	energy
ground state	0^+	0	0	0
Scissors Modes	1^+ 2^+	1 1	0 0	$\hbar\omega + \hbar^2/\mathcal{I}$ $\hbar\omega + 3\hbar^2/\mathcal{I}$
breathing overtone	0^+_1	0	1	$2\hbar\omega$
2^+ overtone	2^+_1	2 0	0 1	$2\hbar\omega + 3\hbar^2/\mathcal{I}$

TABLE I: Quantum numbers of the positive parity states of the Two-Rotor Model.

and n quantum numbers. All the above states however, with the exception of the $J = 2^+$ overtone, are pure K and n states. The ground state 0^+ has quantum numbers $I = \sigma = K = n = 0$, the Scissors Modes $1^+, 2^+$ have quantum numbers $\sigma = 0, K = 1, n = 0$ and $I = 1, 2$ respectively, the breathing overtone, 0^+_1, $I = 0, \sigma = 1, K = 0, n = 1$. The other overtone 2^+_1 has $I = 2, \sigma = 1$ and coupled intrinsic components with $K = 2, n = 0$ and $K = 0, n = 1$.

The collective motion of the $J = 0^+$ overtone is a kind of isovector breathing mode. The $J = 2^+$ overtone is a superposition of the breathing mode in region I (the state φ_{01}), and of the state φ_{20}, which is a relative rotation of the neutron-proton axes as in the Scissors Mode but with angular momentum $K = 2$. Such mixing is determined by the different form that the intrinsic hamiltonian takes in regions I and II. Because of it the $I = 2$ overtone might be called scissors-breathing mode. All these states with their quantum numbers are reported in table 1.

We already mentioned that the spectrum of the intrinsic part of the Two-Rotor Model is identical to that of the planar harmonic oscillator with some constraints due to various symmetries. These constraints reduce the degeneracy of the first excited states of the planar harmonic oscillator from 2 to 1 (the Scissors mode) and of the higher levels form 3 to 2 (the 2 overtones).

Notice that the normalization of the Φ in Eq.(15) is different from that in Ref. [10].

As we said at the beginning, the quadrupole operator to zero order in θ_0 is

$$M(E2,\mu) \;=\; e\,Q_{20}\left[\mathcal{D}^2_{\mu 0}\left(s_I - \frac{1}{2}\,s_{II}\right) + \frac{1}{2}\sqrt{\frac{3}{2}}\left(\mathcal{D}^2_{\mu 2} + \mathcal{D}^2_{\mu -2}\right) s_{II}\right] \qquad (19)$$

where $e\,Q_{20}$ is the quadrupole moment in the intrinsic frame. We then see that while to zero order in θ_0 we cannot excite the breathing overtone from the ground state, we could reach it by the decay of the $J = 2^+$ overtone with the amplitude

$$\langle\Psi_{2M1}|M(E2,\mu)|\Psi_{001}\rangle \;=\; \frac{1}{2\sqrt{10}}\,e\,Q_{20}C^{2M}_{002\mu} \times \left(\langle\varphi_{01}|\varphi_{01}\rangle + \sqrt{3}\langle\phi_{20}|\phi_{01}\rangle\right)$$

where $C^{2M}_{002\mu}$ is a Clebsch-Gordan coefficient. Because

$$\langle\varphi_{01}|\varphi_{01}\rangle = -\sqrt{2}\langle\phi_{20}|\phi_{01}\rangle = \frac{1}{2} \qquad (20)$$

the contributions of the $K = 0$ and $K = 2$ components of the $J = 2^+$ overtone almost cancel out with each other, at variance with the $B(E2, 0^+ \to 2^+_1)$ that is entirely due to its $K = 0$ component, namely its breathing component [4]. Such cancellation appears completely accidental, but it overcompensates the absence of the small factor θ_0^2, so that the transition strength

$$B(E2, 2^+_1 \to 0^+_1) = \frac{1}{32}\left(1 - \sqrt{\frac{3}{2}}\right)^2 e^2\,Q^2_{20} \qquad (21)$$

results negligibly small.

In conclusion the electromagnetic width of the 2^+_1 overtone is entirely due to its $M1$ decay to the Scissors Mode 1^+ and to its $E2$ decay to the ground state [4], and the breathing overtone would remain elusive even if the 2^+_1 overtone were collective enough (and not too much fragmented) to be observed.

[1] N. Lo Iudice and F. Palumbo, Phys. Rev. Lett. **41**, 1532 (1978). The reader interested in the chronology should look at Ref.2 of this paper and at Section 4 of Ref.[6].

[2] D. Bohle, A. Richter, W. Steffen, A. E. L. Dieperink, N. Lo Iudice, F. Palumbo and O. Scholten, Phys. Lett. B **137**, 27 (1984).

[3] J. Enders, H. Kaiser, P. von Neumann-Cosel, C. Rangacharyulu and A. Richter, Phys. Rev. C **59**, R1851 (1999); N. Lo Iudice, Rivista Nuovo Cim. **23**, 1 (2000); K. Heyde, P. von Neumann-Cose and A. Richter, Rev. Mod. Phys. **82** (2010) 2365

[4] K. Hatada. K. Hayakawa and F. Palumbo, Phys. Rev. C **84** (2011) 011302(R)

[5] Y. Sun, C. Wu, K. Bahatt and M. Guidry, Nucl. Phys. A**703** (2002) 130

[6] T. Suzuki and D. W. Rowe, Nucl. phys. **A289** (1977) 461

[7] F. Iachello, Nucl. Phys. **A358** (1981) 89c

[8] E. Lipparini and S. Stringari, Phys. Lett. **B130** (1983) 139

[9] S. G. Rohozinski and W. Greiner, Z. Phys. A**322** (1985) 271

[10] G. De Franceschi, F.Palumbo and N. LoIudice, Phys. Rev. C**29** (1984) 1496

[11] E. Lipparini and S. Stringari, Phys. Rev. Lett. **63**, 570 (1989); H. Portales, E. Duval, and L. Saviot, M. Fujii, M. Sumitomo and S. Hayashi, Phys. Rev. B **63**, 233402 (2001); Ll. Serra, A. Puente, and E. Lipparini, Phys. Rev. B **60**, R13966 (1999); K. Hatada, K. Hayakawa and F. Palumbo, Phys. Rev. B **71**, 092402 (2005).